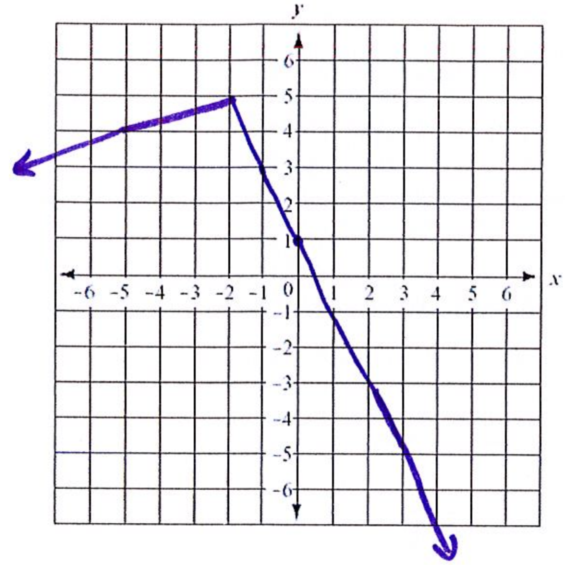
Mr. Michael T. Davis Calculus	Chapter 3 - Test January 23, 2017

Name: Solutions

1. Sketch the graph of a continuous function $y = f(x)$ that has the following properties:

   i. $f(0) = 1$,

   ii. $f'(x) = \begin{cases} -2 \text{ if } x \geq -2 \\ \frac{1}{3} \text{ if } x < -2 \end{cases}$



2. Write an equation for a linear function $y = f(x)$ that has the following properties:

   i. $f(-4) = 3$,

   ii. $f'(-4) = 5$.

$y - 3 = 5(x+4)$

3. The function $f(x) = \begin{cases} x^3 + 4 \text{ if } x > -1 \\ -3x \text{ if } x \leq -1 \end{cases}$ has left-hand and right-hand derivatives at $x = -1$. Does $f(x)$ have a derivative at $x = -1$? Explain why or why not. You may use the derivative rules here.

$(-1)^3 + 4 = -3(-1)$

$-1 + 4 = 3$

$3 = 3$

The function is continuous since

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$

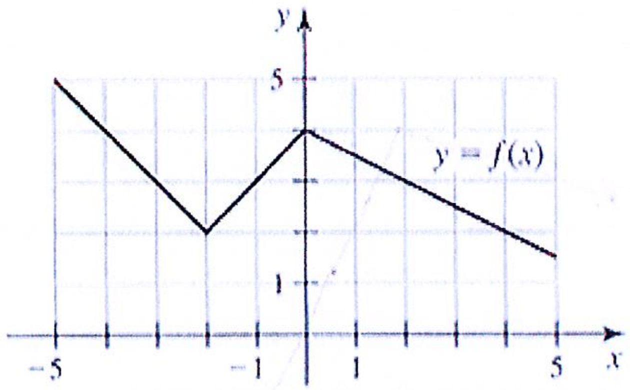
$f'(x) = \begin{cases} 3x^2 \text{ if } x > -1 \\ -3 \text{ if } x < -1 \end{cases}$

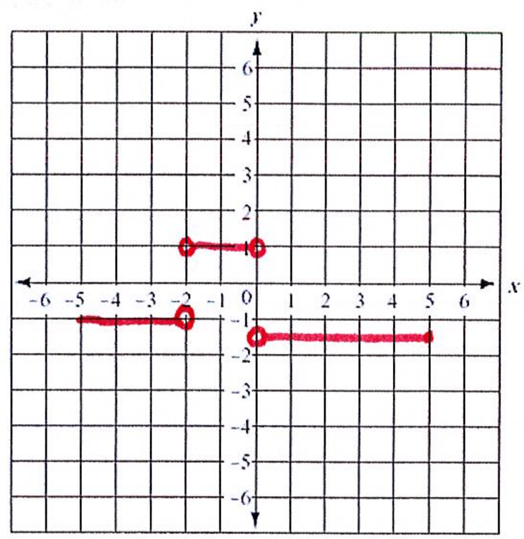
$3(-1)^2 \neq -3$

$+3 \neq -3$

The function $f(x)$ does not have a derivative at $x = -1$.

4. The graph of the function $y = f(x)$ shown here is made of the line segments joined end to end. Graph the function's derivative function.





5. Determine the value(s) of x for which the function $f(x) = \frac{x+2}{\sqrt{x}}$ is not differentiable.

Explain why.

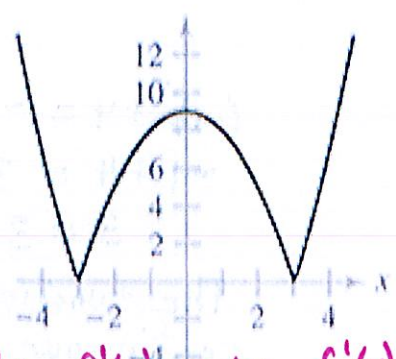
The domain of $f(x)$ is $x > 0$.

$$f'(x) = \frac{\sqrt{x}(1) - (x+2)\frac{1}{2}x^{-\frac{1}{2}}}{x} = \frac{\sqrt{x} - \frac{x+2}{2\sqrt{x}}}{x}$$

$$= \frac{\left(\frac{2x - x - 2}{2\sqrt{x}}\right)}{x} = \frac{x-2}{2x\sqrt{x}}$$ which is not defined on $x \leq 0$ therefore $f(x)$ is differentiable on $x > 0$

6. The graph of a function $y = f(x)$ is shown below. State the values of x for which the function is not differentiable. Explain why.

$f(x)$ is not differentiable at $x = -3$ or $x = 3$ where the graph has cusps and where there is no definitive location for tangent lines. Also,



$\lim\limits_{x \to -3^-} f'(x) \neq \lim\limits_{x \to -3^+} f'(x)$ and $\lim\limits_{x \to 3^-} f'(x) \neq \lim\limits_{x \to 3^+} f'(x)$

7. Find $\frac{dy}{dx}$ for the function $y = x^5 - 2x^3 + 3x^2 - x + 5$

$$\frac{dy}{dx} = 5x^4 - 6x^2 + 6x - 1$$ ✓

8. Find $f'(x)$ for the function $f(x) = 6\sqrt{x} - 4x^{-2} + \frac{2}{x}$

$$f'(x) = \frac{1}{2} \cdot 6x^{-\frac{1}{2}} + 8x^{-3} - 2x^{-2} = \frac{3}{\sqrt{x}} + \frac{8}{x^3} - \frac{2}{x^2}$$

9. Find y' for the function $y = (x+12)^3(\sin x)$

$$y' = 3(x+12)^2 \sin x + (x+12)^3 \cos x$$

10. Find $\frac{dy}{dx}$ for the function $y = \frac{\cos x + 1}{3x}$

$$\frac{dy}{dx} = \frac{3x(\overset{-\sin x}{\cancel{\cos x + 1}}) - 3(\cos x + 1)}{9x^2}$$

11. Write an equation for the line tangent to the graph of $f(x) = x^3 - 2x + 1$ at $x = -2$.

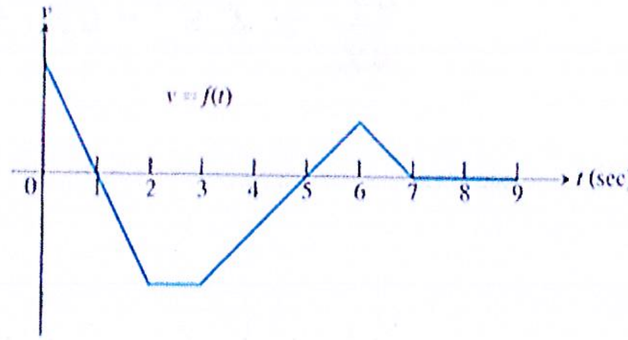
$f(-2) = (-2)^3 - 2(-2) + 1 = -8 + 4 + 1 = -3$

$f'(x) = 3x^2 - 2 \qquad f'(-2) = 3(-2)^2 - 2 = 12 - 2 = 10$

$y - {}^-3 = 10(x - {}^-2)$

$y + 3 = 10(x + 2)$

12. Particle Motion. The accompanying figure shows the velocity $V=f(t)$ of a particle moving on a coordinate number line.



a. When does the particle change direction from right to left? Justify your answer.

at $t=1$ since $v(t)$ changes from positive to negative.

b. When does the particle move backward (to the left)? Justify your answer.

on $(1,5)$ since $V(t)<0$

c. When does the particle slow down? Justify your answer.

on $(0,1)\cup(3,5)\cup(6,7)$ since $|v(t)|$ is decreasing which implies that speed is decreasing.

d. When is the particle's acceleration positive? Justify your answer.

on $(3,6)$ since $V'(t)>0$ or since $V(t)$ is increasing

13. Given $y=2\cos x-\tan x$, determine $\frac{dy}{dx}$

$$\frac{dy}{dx}=-2\sin x-\sec^2 x$$

14. Particle Motion. A particle moves along a real number line (left and right) so that its position at any time $t \geq 0$ is given by the function $s(t) = t^3 - 6t^2 + 9t - 3$ where s is measured in meters and t is measured in seconds. Positive velocity implies movement to the right.

a. Determine the particle's displacement from $t = 0\,\text{sec}$ to $t = 3\,\text{sec}$.

$$s(3) - s(0) = 3^3 - 6(3)^2 + 9(3) - 3 - (0^3 - 6(0)^2 + 9(0) - 3)$$
$$= 27 - 54 + 27 - 3 + 3$$
$$= 0\text{ m}$$

b. Determine the particle's average velocity from $t = 0\,\text{sec}$ to $t = 3\,\text{sec}$.

$$\frac{s(3) - s(0)}{3 - 0} = \frac{0}{3} = 0\text{ m/sec}$$

c. Find the particle's instantaneous velocity at any time t.

$$v(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$$

d. At what time(s) t does the particle change direction. Justify your answer.

$$v(t) = 0 \quad 0 = 3(t^2 - 4t + 3) \quad 0 = 3(t-1)(t-3)$$

$$\begin{array}{ccccccc} & +++ & & --- & & +++ & \\ 0 & & 1 & & 3 & & v \end{array}$$

the particle changes direction at $t = 1$ and $t = 3$ since the sign of $v(t)$ changes

e. Find the particle's acceleration at any time t.

$$a(t) = 6t - 12$$

f. What is the particle's velocity when the acceleration is zero?

$$a(t) = 0 \quad 0 = 6t - 12 \quad 6t = 12 \quad t = 2$$
$$v(2) = 3(2)^2 - 12(2) + 9 = 12 - 24 + 9 = -3\text{ m/sec}^2$$

15. Given $f(x) = 5x^2 + x\sin x$, determine the 2nd derivative, $f''(x)$

$$f'(x) = 10x + \sin x + x\cos x$$
$$f''(x) = 10 + \cos x + \cos x - x\sin x$$
$$= 10 + 2\cos x - x\sin x$$

16. Multiple Choice: Find an equation for the line that is normal to the graph of

$y = \sin x - 2\cos x$ at $x = \dfrac{\pi}{2}$.

a. $y-1=\dfrac{-1}{2}\left(x+\dfrac{\pi}{2}\right)$

b. $y-1=2\left(x-\dfrac{\pi}{2}\right)$

c. $y-1=\dfrac{1}{2}\left(x-\dfrac{\pi}{2}\right)$

d. $y-1=\dfrac{-1}{2}\left(x-\dfrac{\pi}{2}\right)$

e. $y+1=\dfrac{-1}{2}\left(x-\dfrac{\pi}{2}\right)$

$y\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} - 2\cos\frac{\pi}{2}$

$= 1 - 0$

$= 1$

$y'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} + 2\sin\frac{\pi}{2}$

$= 0 + 2$

$= 2$

$y - 1 = -\frac{1}{2}\left(x - \frac{\pi}{2}\right)$